DIMINISHING SENSITIVITY FOR OTHER-REGARDING PREFERENCES

A Senior Honors Thesis

by

SARAH ANNE HILL

Submitted to the Office of Honors Programs
& Academic Scholarships
Texas A&M University
in partial fulfillment of the requirements of the

UNIVERSITY UNDERGRADUATE
RESEARCH FELLOWS

April 2002

Group: Economics and Agribusiness

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ABSTRACT

Diminishing Sensitivity for Other-Regarding Preferences. (April 2002)

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Because of the pervasiveness of diminishing sensitivity in economics, and because diminishing sensitivity has played such a crucial role in examining behavior toward risk, diminishing sensitivity for other-regarding preferences is explored using several proposed models and the equal-division equivalent. By placing restrictions on the models suggested by Fehr and Schmidt and by Charness and Rabin, inequity aversion and diminishing sensitivity can be guaranteed when the player is ahead and behind opponents. There are no simple restrictions that will guarantee diminishing sensitivity in the models suggested by Neilson and Stowe and by Bolton and Ockenfels when the decision-maker is ahead of his opponents. Thus if diminishing sensitivity is a desirable property of a model of other-regarding preferences, the Neilson-Stowe and Bolton-Ockenfels models are not appropriate. For both the Fehr-Schmidt and Charness-Rabin models, the restrictions for diminishing sensitivity imply a dislike of a Robin Hood scheme that redistributes wealth from a higher-payoff opponent to a lower-payoff

opponent. While one interpretation of this result is that diminishing sensitivity is not desirable in the context of other-regarding preferences, the problem actually is with the models themselves. Because the models only consider inequity aversion between the decision-maker and each individual opponent rather than a type of inequity between opponents, these models yield a dislike of redistribution of wealth from high-payoff players to low-payoff players. Therefore these results suggest that perhaps other models should be constructed which allow for diminishing sensitivity and a preference for a redistribution of wealth.

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1. INTRODUCTION

Diminishing sensitivity is a general assumption stating that the further away an object or person is from a source or point of reference, the lower the impact of additional changes. This assumption has had fundamental implications in many areas. In this paper we explore diminishing sensitivity in the context of other-regarding preferences, that is, preferences in which the decision-maker cares about his opponents' payoffs. There are three good reasons for doing so. First, diminishing sensitivity is pervasive in economics, but its implications for other-regarding behavior have not yet been explored. Second, diminishing sensitivity would have been an appropriate starting point for examining preferences in a different context – behavior toward risk. Third, studies by Loewenstein, Thompson, and Bazerman (1989) provide some evidence of diminishing sensitivity for other-regarding preferences.

Diminishing sensitivity has found a place in numerous fields. For example, it provides the basis for models of discounting that now underlie many theories of finance. In economics, it appears in one form as the diminishing marginal rate of substitution. This is the phenomena where, as an individual consumes more and more units of one good, he is willing to give up fewer and fewer units of another good in order to remain at the same utility level. Another example from economics concerns the marginal product, commonly defined as the additional output produced from one additional unit

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This thesis follows the style and format of *Econometrica*.

of input, such as labor. Again because of diminishing sensitivity, the marginal product is generally considered to be diminishing as the number of units of input increases. Diminishing sensitivity is even displayed in physical processes. For instance, the intensities of light and gravity decrease at decreasing rates the greater the distance from their sources.

While the pervasiveness of diminishing sensitivity suggests that it should at least be explored in the new choice context of other-regarding preferences, an analogy with the analysis of behavior toward risk suggests that diminishing sensitivity might be the appropriate place to start when characterizing preferences. When he first introduced what we now know as expected utility theory, Daniel Bernoulli (1738) restricted attention to risk aversion. Risk aversion went on to become the dominant paradigm within expected utility theory until Kahneman and Tversky (1979) provided compelling evidence that while people tend to be risk averse over gains, they also tend to be risk seeking over losses. This more complicated pattern is implied by diminishing sensitivity to changes in wealth. Perhaps diminishing sensitivity can lead to the "right" behavioral assumptions for other-regarding preferences just as it did for preferences toward risk.

The study of other-regarding preferences is a relatively new field in economic theory. Until recently, economists constructed models using the assumption that a person's utility from a competitive game is a function of the individual's own payoff without regard to others' outcomes. However, the data suggest that rather than being entirely selfish, people do care about the payoffs of other players, and so economists are now creating models that are a function of a player's utility from his own payoff as well as his concern for his opponents' payoffs. Evidence of these social preferences can be found in the ultimatum and dictator games. In both games, Player A is asked to divide one dollar between himself and Player B. In the ultimatum game Player B can either accept or reject Player A's proposal. If he accepts, the dollar is divided according to Player A's proposal, and if he rejects, both players receive a payoff of zero. In the dictator game, the dollar is simply divided according to Player A's proposal, and Player B has no input. Standard theory predicts that the outcome in the ultimatum game is that Player A will take $0.99 and give $0.01 to Player B, and Player B accepts this proposal because even one penny is preferred to a payoff of zero. However, experimental evidence shows that in many cases Player B is willing to reject proposals where the offer is relatively small (less than 30 cents), thus indicating a willingness to sacrifice some of his own payoff in order to punish an ungenerous Player A (Thaler (1998)). In the dictator game, standard theory predicts that Player A will take the entire dollar because Player B is unable to reject the proposal. However, Player A often gives at least a small portion of the dollar to Player B (around 20 cents), which indicates that Player A has some *inherent sense* of fairness in dividing the dollar. The results from these two games suggest that people do in fact receive some utility from the payoffs that go to other players. Additionally, research in the area of psychology by Loewenstein, Thompson, and Bazerman (1989) provides some evidence of diminishing sensitivity for other-regarding preferences.

Several models of other-regarding preferences have been proposed, and they differ in how they treat the payoffs of the other players. One model, by Fehr and Schmidt (1999), describes the player's utility as a function of his own earnings and the difference of his opponent's payoff and his own payoff. That is, the model is concerned with how much more (or less) the opponent is earning. In another model by Charness and Rabin (2002), the player's utility is a weighted sum of his own payoff and the other player's earnings. This weight depends upon whether he is doing better or worse than the other player. A model by Bolton and Ockenfels (2000) describes the player's utility as a function of his own payoff as well as his share of the total earnings. This model considers the proportion of the player's earnings relative to the total earnings of all players.

The purpose of this paper is to explore diminishing sensitivity using the above models of other-regarding preferences. We determine what restrictions are necessary in order to guarantee inequity aversion (or a preference for equality in payoffs) and diminishing sensitivity in these models. To do so, we use the equal-division equivalent, or EDE, introduced by Neilson and Stowe (2001). While all of the models are able to show inequity aversion, if diminishing sensitivity is a desirable property, then we are able to specify which models can and cannot exhibit this attribute. After identifying restrictions that imply diminishing sensitivity within the different models discussed above, we go on to examine whether they are desirable restrictions. In particular, we explore whether preferences that satisfy the restrictions exhibit a liking for Robin Hood schemes that redistribute wealth from a higher-payoff opponent to a lower-payoff opponent. We find that the restrictions needed in order for the models to exhibit diminishing sensitivity also create an aversion for this redistribution of wealth, indicating that diminishing sensitivity may not be a desirable property of models of other-regarding preferences, or, perhaps, that the models themselves may not be adequate.

The paper proceeds as follows. Section 2 explains the models and definitions needed for the theorems in section 3. Section 3 presents the three main theorems as well as a corollary. Section 4 explores an alternative interpretation of diminishing sensitivity through the Robin Hood scheme. Section 5 offers a conclusion.

2. PRELIMINARIES

Suppose there is a decision-maker with n partners/opponents. An *allocation* is a vector $(x_0,\dots,x_n) \in R_+^{n+1}$, with $x_0 \geq 0$ the payoff to the decision-maker and $x_i \geq 0$ the payoff to opponent i, for $i = 1,\dots,n$. The decision-maker has preferences over allocations that can be represented by a utility function U: $R_+^{n+1} \to R$.

The utility representation U is meant to be quite general. Other researchers have proposed more specific forms for U, and we wish to leave our construction sufficiently general to include all of them as special cases so as to encompass all the specific forms. In particular, Fehr and Schmidt (1999) assume that U has the form

$$U(x_0,\dots,x_n) = u(x_0) + \sum_{i=1}^{n} v_i(x_i - x_0), \tag{1}$$

so that utility is separable in the decision-maker's own payoff and the differences between his opponents' payoffs and his own payoff. Charness and Rabin (2002) propose a function of the form

$$U(x_0,\dots,x_n) = u(x_0) + \sum_{i=1}^{n} v_i(x_i), \tag{2}$$

so that the utility function is separable in the decision-maker's own payoff and the levels of his opponents' payoffs. Neilson and Stowe (2001) consider a special case of this specification in which

$$U(x_0,\dots,x_n) = \left(1 - \sum_{i=1}^{n} c_i\right) u(x_0) + \sum_{i=1}^{n} c_i u(x_i). \tag{3}$$

In their formulation, the decision-maker uses the same utility function, u, to evaluate everyone's payoff, but weights the utilities differently for different individuals. Finally, Bolton and Ockenfels (2000) specify

$$U(x_0,\ldots,x_n) = u(x_0) + v\left(\frac{x_0}{x_0 + \ldots + x_n}\right), \tag{4}$$

so that utility is separable in the decision-maker's own payoff and his share of the total payoff. This last specification assumes that all payoffs are nonnegative, and that at least one is positive, which is why we restrict attention to allocations in R_+^{n+1}.

To make the ideas of inequity aversion and diminishing sensitivity concrete, we use the concept of an equal-division equivalent proposed by Neilson and Stowe (2001). The *equal-division equivalent*, or *EDE*, of the allocation $(x_0,\ldots,x_n)$ is the value h that satisfies

$$U(x_0,\ldots,x_n) = U(h,\ldots,h). \tag{5}$$

In other words, the equal-division equivalent is set so that the decision-maker is indifferent between the original allocation and everyone receiving the same payoff h. Equation (5) defines a function $h(x_0,\ldots,x_n)$. Since $U(h^*,\ldots,h^*) \geq U(h,\ldots,h)$ for all $h^* \geq h$, the behavior of the function $h(x_0,\ldots,x_n)$ can be used to gauge changes in the decision-maker's utility.

So far, inequity aversion is an ill-defined term. While the basic idea is widely accepted, formal definitions tend to be tied to specific functional forms for the utility function. Neilson and Stowe (2001) define inequity aversion in terms of the equal-division equivalent. We, too, use the equal-division equivalent to formalize inequity aversion, although in a somewhat different way from Neilson and Stowe. Their definition was based on the idea that the decision-maker is made worse off by inequity, and therefore would be willing to sacrifice the group's payoff in order to restore equity. In contrast, we base our definition on the idea that the decision-maker dislikes increases in inequality. More precisely, we say that as x_i moves farther from x_0 for some $i = 1,...,n$, inequality increases, and the decision-maker should dislike the change. Consequently, the EDE should fall.

We say that the utility function U exhibits *inequity aversion when ahead* if $\partial h/\partial x_i \geq 0$ for all $i = 1,...,n$ and all $x_i < x_0$. If $x_i < x_0$, the decision-maker is ahead of opponent i in the sense that his payoff is higher than i's. An increase in x_i reduces inequality by making opponent i's payoff closer to the decision-maker's, which should make the EDE rise. Similarly, we say that U exhibits *inequity aversion when behind* if $\partial h/\partial x_i \leq 0$ for all $i = 1,...,n$ and all $x_i > x_0$. When $x_i > x_0$, an increase in x_i moves opponent i farther ahead of the decision-maker, which should make EDE fall. The utility function exhibits *inequity aversion* if it exhibits inequity aversion when both ahead and behind. Finally, we say that inequity aversion (when ahead or behind) is *strict* if the partial derivative $\partial h/\partial x_i$ is non-zero and has the appropriate sign.

Diminishing sensitivity suggests that the farther something gets from a reference point, the less changes should matter. Here we look at the decision-maker's sensitivity to changes in his opponents' payoffs, and we use as a reference point the decision-maker's own payoff. This reference point makes sense because if an opponent's payoff is at the reference point, his payoff and the decision-maker's are equal, and, at least within the two-person subgroup, there is no inequity. Any movement in the opponent's payoff away from the reference point increases inequality, which an inequity averse decision-maker is dislikes. Diminishing sensitivity suggests that the farther the opponent's payoff moves from the inequity averse decision-maker's payoff, the less additional movements reduce the decision-maker's utility.

To formalize this notion, we say that a utility function U that is inequity averse when ahead exhibits *diminishing sensitivity when ahead* if $\partial^2 h/\partial x_i^2 \geq 0$ when $x_i < x_0$. When $x_i < x_0$, an increase in x_i moves opponent i's payoff closer to the reference point. According to diminishing sensitivity, movements away from the reference point should be accompanied by smaller changes in utility, which we measure as smaller changes in the EDE h. So, as x_i moves closer to x_0 from below, additional increases in x_i should lead to larger changes in the EDE. Similarly, a utility function that is inequity averse when behind exhibits *diminishing sensitivity when behind* if $\partial^2 h/\partial x_i^2 \geq 0$ when $x_i > x_0$. When the decision-maker is behind, increases in x_i represent movements away from the reference point. They also represent increases in inequality, and therefore a decline in h. As x_i moves farther from the reference point, the impact of additional movements should be reduced, meaning that the corresponding changes in the EDE should be smaller, that is, less negative. Consequently, the derivative $\partial^2 h/\partial x_i^2$ should be positive. We say that an inequity averse utility function exhibits *diminishing sensitivity* if it exhibits it when both ahead and behind. Finally, we say that diminishing sensitivity (when ahead or behind) is *strict* when $\partial^2 h/\partial x_i^2 > 0$.

3. INEQUITY AVERSION AND DIMINISHING SENSITIVITY

In this section, we examine four different functional forms for the utility function to determine what restrictions, if any, guarantee inequity aversion and diminishing sensitivity. The functional forms are chosen to reflect some other-regarding utility functions already present in the literature. We first look at a functional form consistent with that proposed by Fehr and Schmidt (1999), then at one proposed by Charness and Rabin (2002) and a special case examined by Neilson and Stowe (2001), and finally a form consistent with that investigated by Bolton and Ockenfels (2000).

Throughout this section we restrict attention to the setting in which the decision-maker has a single opponent. The results can be easily extended to settings with more opponents. In all cases considered below, the utility function U over two-person allocations has the form $U(x_0,x_1) = u(x_0) + v(f(x_0,x_1))$. For allocations among more players, one could use the utility function $U(x_0,\ldots,x_n) = u(x_0) + v_1(f_1(x_0,\ldots,x_n)) + \ldots + v_n(f_n(x_0,\ldots,x_n))$, and the restrictions found on the function v below for the single-opponent setting could be applied to each of the n functions $v_1,\ldots,v_n$.

We begin with a functional form consistent with the one proposed by Fehr and Schmidt (1999). They analyze the utility function $U(x_0,x_1) = u(x_0) + v(x_1 - x_0)$, so that utility depends on the decision-maker's own payoff and the difference between his opponent's payoff and his own payoff.

Theorem 1. Suppose $U(x_0,x_1) = u(x_0) + v(x_1 - x_0)$ with $u' > 0$ and $u'' < 0$. Then this function exhibits inequity aversion when ahead if $v'(z) \geq 0$ when $z < 0$ and inequity aversion when behind if $v'(z) \leq 0$ when $z > 0$. It satisfies diminishing sensitivity if, in addition, $v'' \geq 0$ except at 0.

Proof. By definition,

$$u(h(x_0, x_1)) = u(x_0) + v(x_1 - x_0). \tag{6}$$

This implies

$$\frac{\partial h}{\partial x_1} = \frac{v'(x_1 - x_0)}{u'(h(x_0, x_1))}. \tag{7}$$

So when $z < 0$, $\partial h/\partial x_1 \geq 0$, and when $z > 0$, $\partial h/\partial x_1 \leq 0$. Therefore, $U(x_0, x_1)$ exhibits inequity aversion.

Further differentiation of equation (7) yields

$$\frac{\partial^2 h}{\partial x_1^2} = \frac{u'(h(x_0, x_1))v''(x_1 - x_0) - v'(x_1 - x_0)u''(h(x_0, x_1))\dfrac{\partial h}{\partial x_1}}{(u'(h(x_0, x_1)))^2}. \tag{8}$$

Since $u' > 0$, $v' \cdot (\partial h/\partial x_1) \geq 0$, and $u'' < 0$, if $v'' \geq 0$, then $\partial^2 h/\partial x_1^2 \geq 0$. Thus $U(x_0, x_1)$ exhibits diminishing sensitivity. ■

For Theorem 1, the assumptions of $u' > 0$ and $u'' < 0$ follow the standard theory so that as the decision-maker's payoff increases, his utility increases at a decreasing rate (that is, the decision-maker shows diminishing sensitivity for his own payoffs). When $x_1 < x_0$, the decision-maker is ahead, so an increase in the opponent's payoff means that the difference in the payoffs (x_1 - x_0) has decreased and is now closer to zero. Thus the decision-maker's utility increases as a result of the equalization in payoffs, and $v'(z) \geq$

0. On the other hand, when $x_1 > x_0$, the decision-maker is behind, so an increase in the opponent's payoff means that the difference in the payoffs (x_1 - x_0) has increased and has moved further away from zero. Thus the decision-maker's utility decreases as a result of greater inequity in the payoffs, and $v'(z) \leq 0$.

Additionally, $v''(z) > 0$ means that as x_1 moves away from x_0, the decision-maker exhibits diminishing sensitivity with respect to the difference in the payoffs. This can be shown by graphing $v(z)$ against $z = x_1 - x_0$ (see Figure 1). When $z > 0$, as the difference in the payoffs increases, the decision-maker's utility decreases at a decreasing rate, giving $v''(z) \geq 0$. When $z < 0$, as the difference in payoffs increases, the decision-maker's utility decreases at a decreasing rate, again giving $v''(z) \geq 0$. Thus $v''(z) \geq 0$ for all z except at 0.

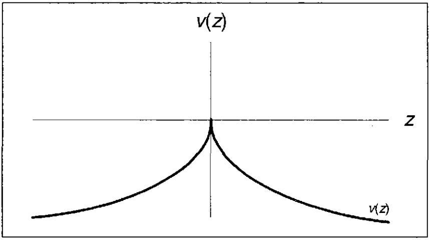

FIGURE 1.-Fehr-Schmidt utility function.

We now turn attention to a second functional form for the other-regarding utility function, the form proposed by Charness and Rabin (2002). Where Fehr and Schmidt assumed that the opponent's payoff enters the decision-maker's utility function only as the difference between the two payoffs, Charness and Rabin assume that the level of the opponent's payoff matters, so that $U(x_0,x_1) = u(x_0) + v(x_1)$.

Theorem 2. Suppose $U(x_0,x_1) = u(x_0) + v(x_1)$ with $u' > 0$, $u'' < 0$, and $u'(x) > |v'(x)|$ for all x. Then this function exhibits inequity aversion when ahead if $v'(x_1) \geq 0$ for $x_1 < x_0$, and exhibits inequity aversion when behind if $v'(x_1) \leq 0$ for $x_1 < x_0$. It satisfies diminishing sensitivity if, in addition, $0 \leq v''(x) \leq -u''(x)$ for all x except at 0.

Proof. By definition,

$$u(h(x_0,x_1)) + v(h(x_0,x_1)) = u(x_0) + v(x_1). \tag{9}$$

This implies

$$\frac{\partial h}{\partial x_1} = \frac{v'(x_1)}{u'(h(x_0,x_1)) + v'(h(x_0,x_1))}. \tag{10}$$

So when $x_1 < x_0$, $\partial h/\partial x_1 \geq 0$, and when $x_1 > x_0$, $\partial h/\partial x_1 \leq 0$. Therefore, $U(x_0, x_1)$ exhibits inequity aversion.

Further differentiation of equation (10) yields

$$\frac{\partial^2 h}{\partial x_1^2} = \frac{1}{(u'(h(x_0,x_1)) + v'(h(x_0,x_1)))^2}\Big[(u'(h(x_0,x_1)) + v'(h(x_0,x_1)))v''(x_1) \\ - v'(x_1)\left(u''(h(x_0,x_1))\frac{\partial h}{\partial x_1} + v''(h(x_0,x_1))\frac{\partial h}{\partial x_1}\right)\Big]. \tag{11}$$

Because $u' > 0$ and $u' > |v'|$, if $v'' > 0$, then $[u'(h(x_0,x_1)) + v'(h(x_0,x_1))]v''(x_1) \geq 0$. The only part of equation (11) left to examine is

$$v'(x_1)\left(u''(h(x_0,x_1))\frac{\partial h}{\partial x_1} + v''(h(x_0,x_1))\frac{\partial h}{\partial x_1}\right).$$

Because $\partial h/\partial x_1 = v'(x_1)/(u'(h) + v'(h))$, the above term can be written as

$$\frac{v'(x_1)^2\left(u''(h(x_0,x_1)) + v''(h(x_0,x_1))\right)}{u'(h(x_0,x_1)) + v'(h(x_0,x_1))}$$

Since $u' > 0$, $u' > |v'|$, $u'' < 0$, and $-u'' \geq v''$, the above term is then non-positive and $\partial^2 h/\partial x_1^2 \geq 0$. Thus $U(x_0, x_1)$ exhibits diminishing sensitivity. ■

For Theorem 2, when $x_1 < x_0$, the decision-maker is ahead, so an increase in the opponent's payoff means that the difference in the payoffs has decreased. Thus the decision-maker's utility increases as a result of the equalization in payoffs, and $v' \geq 0$. On the other hand, when $x_1 > x_0$, the decision-maker is behind, so an increase in the opponent's payoff means that the difference in the payoffs has increased. Thus the decision-maker's utility decreases as a result of greater inequity in the payoffs, and $v' \leq 0$. The requirements that $u' > |v'|$ and $-u'' > v''$ make sense in this context because an individual normally gives more weight to his own payoff than his opponent's payoff.

Additionally there is the condition that $v'' \geq 0$, which while possible, is rather restrictive. In order for all of the above restrictions to be true, when $v' \geq 0$ this means that the functions u and v must be asymptotic to parallel lines (see Figure 2), or the domain must be bounded. In the case where $v' \leq 0$, v is below u and either asymptotic to a line with a negative slope (see Figure 2), or the domain must be bounded.

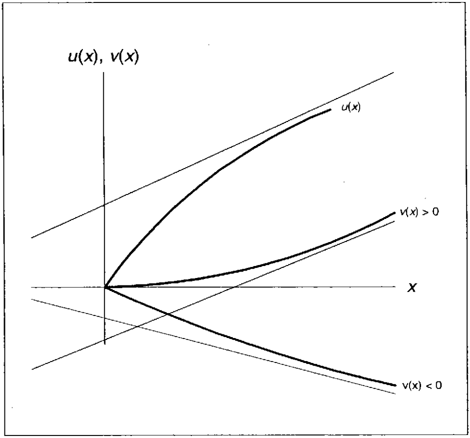

FIGURE 2.-Charness-Rabin utility function.

With this model it is important to note the restrictions created by the assumptions for inequity aversion and diminishing sensitivity. For the function v, it is necessary to assume that that $v' \geq 0$ when $x_1 < x_0$ and $v' \leq 0$ when $x_1 > x_0$. Thus while v

is a function of x_1, it is also implicitly a function of x_0. This result suggests that to accommodate inequity aversion, v should be explicitly written as a function of x_0 and x_1.

Furthermore, the conditions that $-u'' > v''$ and $v'' \geq 0$ greatly restrict both u and v. To guarantee that $v'' \geq 0$, it must be true that $u'(x_0) \geq v'(x_1)$ for all x_0 and x_1. While this is possible, as shown by Figure 2, these limitations may not be desirable for a model of other-regarding preferences.

Neilson and Stowe (2001) use an axiomatic approach to construct a model that can be considered a special case of the Charness-Rabin model. Their axioms imply the existence of a utility function with the form $U(x_0,x_1) = (1-c)u(x_0) + cu(x_1)$, which can be seen to be a special case of Charness and Rabin's formulation when $v(x_1) = cu(x_1)/(1-c)$. Consequently, the restrictions needed on the Neilson-Stowe utility function can be found by applying Theorem 2.

Corollary. Suppose $U(x_0,x_1) = (1-c)u(x_0) + cu(x_1)$ with $u' > 0$ and $u'' < 0$. Then this function exhibits inequity aversion when ahead if $0 \leq c < ½$, and exhibits inequity aversion when behind if $c \leq 0$. It satisfies diminishing sensitivity if $c \leq 0$.

Proof. The function from Theorem 2 can be written as $U(x_0, x_1) = W(x_0) + V(x_1)$ in order to differentiate between that form and this special case. Theorem 2 then implies that $W = (1 - c)u$ and $V = cu = [c/(1-c)]W$. This implies $W' = (1 - c)u'$ and $V' = [c/(1-c)]W'$. Also, $W'' = (1 - c)u''$ and $V'' = [c/(1-c)]W''$. Theorem 2 requires that $u'(x) > |v'(x)|$ for all x, which holds if $c < ½$. If $u' > 0$ and $u'' < 0$, then according to

Theorem 2, U exhibits inequity aversion when ahead if $V' \geq 0$, which requires $0 \leq c < \frac{1}{2}$, and U exhibits inequity aversion when behind if $V' \leq 0$, which requires $c \leq 0$.

Similarly, according to Theorem 2, U exhibits diminishing sensitivity when $0 \leq V' \leq -W'$, which occurs when $c \leq 0$. ■

There are several points to note with the Corollary to Theorem 2. First, the results for inequity aversion show that when the decision-maker is ahead, the constant c is positive (although less than ½), and when the decision-maker is behind, c is negative. These assumptions seem appropriate in the context of this model, although they require that c depend on x_0 and x_1. When the decision-maker is ahead, $0 \leq c < \frac{1}{2}$, so the decision-maker's utility increases as a result of gains in his own payoff as well as his opponent's, but he gives greater weight to his own payoff. When the decision-maker is behind, $c \leq 0$, so his utility increases as a result of gains in his own payoff and decreases as a result of gains in his opponent's payoff.

The primary problem with the functional form is that it cannot exhibit diminishing sensitivity when c is positive. Since $c \geq 0$ is the condition for inequity aversion when ahead, the decision-maker cannot exhibit both inequity aversion and diminishing sensitivity when ahead. Consequently, if one views diminishing sensitivity when ahead as a valid requirement of the other-regarding utility function, the Neilson-Stowe model fails.

The final functional form to be considered here is the one proposed by Bolton and Ockenfels (2000). In their formulation, utility depends on the level of the decision-maker's payoff and his share of the total, so that $U(x_0, x_1) = u(x_0) + v(x_0/(x_0 + x_1))$.

Theorem 3. Suppose $U(x_0,x_1) = u(x_0) + v\left(\frac{x_0}{x_0+x_1}\right)$ with $u' > 0$, $u'' < 0$. Then this model exhibits inequity aversion when ahead if $v'(z) \leq 0$ for $z > ½$ and inequity aversion when behind if $v'(z) \geq 0$ for $z < ½$. It satisfies diminishing sensitivity when behind, if, in addition, $v''(z) \geq 0$ except at ½.

Proof. By definition,

$$u(h(x_0,x_1)) + v\left(\frac{1}{2}\right) = u(x_0) + v\left(\frac{x_0}{x_0+x_1}\right). \tag{12}$$

This implies

$$\frac{\partial h}{\partial x_1} = \frac{-\left(\frac{x_0}{(x_0+x_1)^2}\right)v'\left(\frac{x_0}{x_0+x_1}\right)}{u'(h(x_0,x_1))}. \tag{13}$$

So when $x_1 < x_0$, $\partial h/\partial x_1 \geq 0$, and when $x_1 > x_0$, $\partial h/\partial x_1 \leq 0$. Therefore, $U(x_0, x_1)$ exhibits inequity aversion.

Further differentiation of equation (13) yields

$$\begin{aligned}\frac{\partial^2 h}{\partial x_1^2} = \frac{1}{(u'(h(x_0,x_1)))^2}&\left[\left[u'(h(x_0,x_1))\left[\frac{x_0}{(x_0+x_1)^3}v'\left(\frac{x_0}{x_0+x_1}\right) + \frac{x_0}{(x_0+x_1)^4}v''\left(\frac{x_0}{x_0+x_1}\right)\right]\right]\right.\\ &\left.+\left[\frac{x_0}{(x_0+x_1)^2}v'\left(\frac{x_0}{x_0+x_1}\right)(u''(h(x_0,x_1))\frac{\partial h}{\partial x_1}\right]\right].\end{aligned} \tag{14}$$

Since $u' > 0$, $u'' < 0$, and $v' \geq 0$ and $\partial h/\partial x_1 \leq 0$ when $x_1 > x_0$, if $v'' \geq 0$, then $\partial^2 h/\partial x_1^2 >$ 0. Thus $f(x_0, x_1)$ exhibits diminishing sensitivity when behind. ∎

Note that in Theorem 3 a function which exhibits inequity aversion may not exhibit diminishing sensitivity when $x_1 < x_0$. For example, if $v'' > 0$ in equation (14), then it is possible to have

$$u'(h(x_0,x_1))\left[\frac{x_0}{(x_0+x_1)^3}v'\left(\frac{x_0}{x_0+x_1}\right)+\frac{x_0}{(x_0+x_1)^4}v''\left(\frac{x_0}{x_0+x_1}\right)\right]$$
$$+\left[\frac{x_0}{(x_0+x_1)^2}v'\left(\frac{x_0}{x_0+x_1}\right)(u''(h(x_0,x_1))\frac{\partial h}{\partial x_1}\right]<0,$$

giving $\partial^2 h/\partial x_1^2 < 0$. Similarly, if $v'' < 0$, it may be that $\partial^2 h/\partial x_1^2 < 0$.

For Theorem 3, when $x_1 < x_0$, the decision-maker is ahead, so an increase in the opponent's payoffs means that the difference in the payoffs has decreased. Thus the decision-maker's utility increases as a result of this equalization in payoffs, however, the decision-maker's share of the payoffs has actually decreased, therefore $v' \leq 0$. On the other hand, when $x_1 > x_0$, the decision-maker is behind, so an increase in the opponent's payoff means that the difference in the payoffs has increased. Thus the decision-maker's utility decreases as a result of greater inequity in the payoffs, and since the decision-maker's share of payoffs has also decreased, $v' \geq 0$.

Additionally, $v'' \geq 0$ means that as x_1 moves away from x_0, the decision-maker exhibits diminishing sensitivity with respect to his share of payoffs. This can be shown by graphing v against the decision-maker's share of payoffs (see Figure 3). When the share of payoffs is less than ½, the decision-maker is behind, and so as the inequity in payoffs increases and the share of payoffs becomes smaller, the decision-maker's utility will decrease at a decreasing rate. Thus $v'' \geq 0$ when the decision-maker is behind ($x_1 > x_0$).

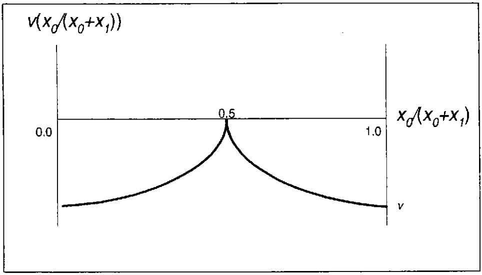

FIGURE 3.-Bolton-Ockenfels utility function.

Table I summarizes the results of the three theorems. All three models are able to accommodate inequity aversion, although the Charness-Rabin model requires that the function v depend on both x_1 and x_0. Also, all three models are able to handle diminishing sensitivity when behind. They differ in their ability to accommodate diminishing sensitivity when ahead; in particular, the Bolton-Ockenfels model and the Neilson-Stowe model are unable to guarantee diminishing sensitivity when ahead.

TABLE I
SUMMARY OF RESULTS

	Fehr – Schmidt	**Charness – Rabin**	**Bolton – Ockenfels**
	$u(x_0) + v(x_1 - x_0)$	$u(x_0) + v(x_1)$	$u(x_0) + v(x_0/(x_0 + x_1))$
Inequity Aversion when Ahead	$v' > 0$	$v' > 0$	$v' < 0$
Inequity Aversion when Behind	$v' < 0$	$v' < 0$	$v' > 0$
Diminishing Sensitivity when Ahead	$v'' > 0$	$v'' > 0$	---
Diminishing Sensitivity when Behind	$v'' > 0$	$v'' > 0$	$v'' > 0$

4. AN ALTERNATIVE INTERPRETATION OF DIMINISHING SENSITIVITY

In the preceding section, we quantified diminishing sensitivity in terms of its implications for changes in the EDE in a setting with one opponent. There are, of course, other ways to think about diminishing sensitivity. A natural one is to think about how the decision-maker compares two different opponents, one of whose payoffs is farther from the decision-maker's than the other's. Intuition tells us that the decision-maker would be made strictly better off by taking money away from the richer of the two opponents and giving it to the poorer. As we show in this section, these intuitively plausible preferences are not always consistent with diminishing sensitivity.

Consider the following scenario. The decision-maker has two opponents, and his preferences are represented by the function $U(x_0,x_1,x_2)$. Assume, without loss of generality, that $x_2 > x_1$. We are interested in the following question: Would the decision-maker prefer to take a (marginal) amount of money away from opponent 2 and give it to opponent 1?

Begin with the Fehr-Schmidt specification from Theorem 1, where the decision-maker's utility function is $U(x_0,x_1,x_2) = u(x_0) + v(x_1 - x_0) + v(x_2 - x_0)$. Consider a Robin Hood scheme in which the allocation changes so that player 1's payoff is increased by the marginal amount dx and player 2's is decreased by the same amount. The decision-maker's utility then changes by

$$dU = v'(x_1 - x_0)dx - v'(x_2 - x_0)dx.$$

By Theorem 1, diminishing sensitivity holds if $v'' \geq 0$ except at zero, which in turn implies that $dU \leq 0$ under the Robin Hood scheme as long as the decision-maker is either ahead or behind both opponents. If $x_1 < x_0 < x_2$, and if the Theorem 1 requirements for inequity aversion hold, $dU \geq 0$. Still, though, in the Fehr-Schmidt setting, diminishing sensitivity and a preference for reallocating from rich to poor are incompatible.

The Charness-Rabin specification from Theorem 2 is similar. Here the decision-maker's utility function is $U(x_0, x_1, x_2) = u(x_0) + v(x_1) + v(x_2)$. Using the Robin Hood scheme again, the decision-maker's utility changes by

$$dU = v'(x_1)dx - v'(x_2)dx.$$

By Theorem 2, diminishing sensitivity holds if $v'' \geq 0$ except at zero, and so $dU \leq 0$ when the decision-maker is ahead or behind both opponents. If $x_1 < x_0 < x_2$, and if the Theorem 2 requirements for inequity aversion hold, $dU \geq 0$. Thus in the Charness-Rabin model diminishing sensitivity and a preference for reallocating from rich to poor are again incompatible.

An inspection of the Bolton-Ockenfels specification gives a different result. Here the player only looks at his share of the total payoffs, and so he is indifferent as to how the marginal amount dx is allocated between player 1 and player 2 in the Robin Hood scheme. As long as x_0 and the total payoff to all of the players do not change, then the decision-maker's utility does not change. So in the Bolton-Ockenfels model, diminishing sensitivity indicates an indifference for the Robin Hood scheme.

It is worthwhile to note here why diminishing sensitivity and reallocating from rich to poor are incompatible in the Fehr-Schmidt and Charness-Rabin specifications. The changes in utility due to the Robin Hood scheme are a result of inequity aversion, and the degrees of the changes in utility are a result of diminishing sensitivity.

Consider, for example, the case where $x_0 < x_1 < x_2$. When player 1's payoff increases by dx and player 2's payoff decreases by dx, then the inequity between the decision-maker and player 1 increases while the inequity between the decision-maker and player 2 decreases. Because of diminishing sensitivity, however, the overall utility decreases because the player is more sensitive to the change in player 1's payoff. Since the decision-maker's utility is not directly a function of the difference in the payoffs of players 1 and 2, then any change in utility from the decrease in inequity between the payoffs of players 1 and 2 is not accounted for by these two models. The results are similar when $x_1 < x_2 < x_0$.

Finally there is the case where $x_1 < x_0 < x_2$. In this instance when player 1's payoff increases by dx and player 2's payoff decreases by dx, then the inequity between the decision-maker and player 1 decreases and the inequity between the decision-maker and player 2 also decreases. Because there is less inequity between the decision-maker's payoff and the payoffs to player 1 as well as player 2, the decision-maker's utility increases. Again, however, this change in utility is not the result of a decrease in inequity of the payoffs of players 1 and 2.

While the Fehr-Schmidt and Charness-Rabin specifications seem to have the property that diminishing sensitivity and the Robin Hood scheme are incompatible, this may actually be due to the construction of the models themselves. Both specifications base the decision-maker's utility function upon inequity aversion only with opponents, and neither considers a direct type of inequity aversion based upon differences in the opponents' payoffs.

5. CONCLUSION

Because of the pervasiveness of diminishing sensitivity in economics, and because diminishing sensitivity has played such a crucial role in examining behavior toward risk, we have explored diminishing sensitivity for other-regarding preferences using several proposed models and the EDE. We found that by placing restrictions on the models suggested by Fehr and Schmidt and by Charness and Rabin, we can guarantee inequity aversion and diminishing sensitivity when the player is ahead and behind. On the other hand, the Neilson-Stowe model does not exhibit diminishing sensitivity when the player is ahead, and there are no simple restrictions that will guarantee diminishing sensitivity in the Bolton-Ockenfels model when the player is ahead. Thus if diminishing sensitivity is a desirable property of a model of other-regarding preferences, the Neilson-Stowe and Bolton-Ockenfels models are not appropriate.

In order to determine whether diminishing sensitivity is desirable in these models, we looked at a Robin Hood scheme that redistributes wealth from a higher-payoff opponent to a lower-payoff opponent. For both the Fehr-Schmidt and Charness-Rabin models, the restrictions for diminishing sensitivity imply a dislike of this Robin Hood scheme. While one interpretation of this result is that diminishing sensitivity is not desirable in the context of other-regarding preferences, the problem actually is with the models themselves. Because the models only consider inequity aversion between the player and each individual opponent rather than a type of inequity aversion for the payoffs to all players, these models yield a dislike of the Robin Hood scheme.

Therefore, these results suggest that perhaps another model should be constructed which allows for diminishing sensitivity and a preference for a redistribution of wealth. One such model is as follows. Define $x_L = \min\{x_i \mid x_i < x_0\}$ and $x_H = \max\{x_i \mid x_i > x_0\}$. Then let $U(x_0,\ldots,x_n) = u(x_0) + v_L(x_L - x_0) + v_H(x_H - x_0)$. Notice that if the player has only one opponent, then this model is the same as the Fehr-Schmidt model for which we have already characterized inequity aversion and diminishing sensitivity. Also, here the player will have a preference for some Robin Hood schemes because the player's utility will increase as the wealth of the lowest-payoff opponent increases, and similarly the player's utility will increase as the wealth of the highest-payoff opponent decreases. It is likely that there are many other models that could also capture diminishing sensitivity for other-regarding preferences and a preference for the redistribution of wealth.

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